Reg. No. : ....

de No. : 5762 Sub. Code : WMAM 23

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2024.

Second Semester

Mathematics — Core

PARTIAL DIFFERENTIAL EQUATIONS

(For those who joined in July 2023 onwards)

e : Three hours Maximum : 75 marks

PART A — ($15 \times 1 = 15$ marks)

Answer ALL the questions.

Choose the correct answer :

The Poisson equation is ______________.

(a) $\nabla^2 u = f(x, y, z)$

(b) $u_{t_t} + c^2 \nabla^4 u = 0$

(c) $\nabla^4 u = \nabla^2 (\nabla^2 u) = 0$

(d) $\nabla^2 u + \lambda u = 0$

2. If the canonical form of the equation ______________, then the general solution can immediately ascertained.

(a) General (b) Simple

(c) Original Solution (d) Zero

3. If $B^2 - 4AC > 0$, the equation is of ______________ type.

(a) Parabolic (b) Elliptic

(c) Hyperbolic (d) None of the above

4. Determine the solution of initial value problem

$u_{tt} - c^2 u_{xx} = x$, $u(x, 0)$, $u_t(x, 0) = 3$.

(a) $u(x, t) = x + \left(\frac{1}{c}\right) \sin x \sin ct$

(b) $u(x, t) = 3t + \frac{1}{2} x t^2$

(c) $u(x, t) = t$

(d) $u((x, t) = \cos x \cos ct + t)$

5. Determine the solution of initial value problem

$u_{tt} - c^2 u_{xx} = x$, $u(x, 0) = x^3$, $u_t(x, 0) = x$

(a) $u\left((x, t) = \cos x \cos ct + \left(\frac{t}{c}\right)\right)$

(b) $u\left((x, t) = \cos ct + \left(\frac{t}{c}\right)\right)$

(c) $u(x, t) = x^3 + 3c^2 x t^2 + xt$

(d) $u(x, t) = \sin x \cos ct + x^2 t + \frac{1}{3} c^2 t^3$

The D'Alembert solution has the form __________.

(a) $u(x, t)=\frac{1}{2}[f(x+ct)+f(x-ct)]$

(b) $u_t - k(u_{x_x} + u_{y_y} + u_{z_z}) = 0$

(c) $u(x, t)=[f(x+ct)+f(x-ct)]$

(d) $u(x, t)=\frac{1}{2}[f(x+ct)f(x-ct)]$

7. The Lapalce transformation of $1/\sqrt{t}$ is

(a) $\frac{1}{s}-a$

(b) $\left(\frac{1}{s-a}\right)$

(c) $s-a$

(d) $\sqrt{\frac{\pi}{8}}$

8. The Laplace transformation of $1/\sqrt{\pi t}(1+2at)e^{at}$ is

(a) $\frac{s}{s-a\sqrt{s-a}}$

(b) $\frac{a}{s-a}$

(c) $\frac{1}{\sqrt{s+a}}$

(d) $s-a$

9. Which boundary condition $\left(\frac{\partial u}{\partial n}\right)$ is prescribed at a boundary?

(a) Neumann condition

(b) Mixed condition

(c) Dirichlet condition

(d) All (a), (b), (c)

10. Schrodinger equation (time dependent) ________

(a) $\nabla^2 u + [k - g(\phi)]u = 0$

(b) $\nabla u + g(\phi) = 0$

(c) $\nabla^2 u + [k - g(\phi)]u = 0$

(d) $\nabla^2 u + ku = 0$

11. In free space the Poisson equation become

(a) Maxwell equation

(b) Ampere equation

(c) Laplace equation

(d) Steady state equation

Which one of the following is a property of the solutions to the Laplace equation : $\nabla^2 f = 0$?

(a) The solution have neither maxima nor minima anywhere except at the boundaries

(b) The solution are not separable in the coordinates

(c) The solution are not continuous

(d) The solution are not dependent on the boundary condition

The Greens function is ________.

(a) Antisymmetric (b) Symmetric

(c) Operators (d) Delta

Let us consider the differential equation $\frac{d^2y}{dx^2} = f(x)$, $y(0) = y(1) = 0$ if $G(x, s)$ be the greens function. Then which of the following is correct?

(a) $G(x, s) = G(s, x)$ for all x and s

(b) $G(x, s) = G(s, x)$ for some x and s

(c) $G(x, s) = -G(s, x)$ for all x and s

(d) $G(x, s) = -G(s, x)$ for some x and s

15. The Dirac delta function is also known

________________.

(a) Impulse (b) Greens function

(c) Unit Dirac (d) Unit impluse

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the equation $u_{xx} + x^2 u_{yy} + 0$ elliptic.

Or

(b) Explain hyperbola type in equation wi constant co-efficient.

17. (a) Determine the solution of the initial valu problem $u_{tt} - c^2 u_{xx} = 0$, $u(x, 0) = \sin x$ $u_t(x, 0) = x^2$.

Or

(b) Solve the initial value problem $u_{xx} + 2u_{xy} - 3u_{yy} = 0$, $u(x, 0) = \sin x$, $u_y(x, 0) = x$

18. (a) Solve $u_{tt} = c^2 u_{xx}$, $0 < x < 1$, $t > 0$,

$u(x, 0) = x(1 - x)$, $\quad u_t(x, 0) = 0$, $\quad 0 \le x \le 1$,
$u(0, t) = u(1, t) = 0$, $t > 0$.

Or

(b) Obtain the solution $u_t = 4u_{xx}$, $0 < x < 1$, $t > 0$,
$u(x, 0) = x^2(1 - x)$, $\quad 0 \le x, \le 1$, $\quad u(0, t) = 0$,
$u(l, t) = 0$, $t \ge 0$.

19. (a) Determine the solution $\nabla^2 u = 0$, $1 < r < 2$, $0 < \theta < \pi$,
$u(1, \theta) = \sin\theta$, $u(2, \theta) = 0$, $0 \le \theta \le \pi$,
$u(r, 0) = 0$, $u(r, \pi) = 0$, $1 \le r \le 2$.

Or

(b) Solve $\nabla^2 u = 0$, $1 < r < 2$, $0 < \theta < 2\pi$,

$u_r(1, \theta) = \sin\theta$, $u_r(2, \theta) = 0$, $0 \le \theta \le 2\pi$.

20. (a) Obtain the solution of Laplace equation
$\nabla^2 u = 0$, $\quad 0 < r < \infty$, $\quad 0 < \theta < 2\pi$,
$u(r, 0+) = u(r, 2\pi -) = 0$.

Or

(b) Prove that $\frac{\partial G}{\partial n}$ is discontinuous at (ξ, η); in particular

$\lim_{\epsilon \to 0} \iint_{C_\varepsilon} \frac{\partial G}{\partial n} ds = 1$, $C_\varepsilon : (x - \xi)^2 + (y - \eta)^2 = \epsilon^2$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL the questions, choosing either (a) or (b).

21. (a) Consider the current $I(x, t)$ and the potentia $v(x, t)$ at a point x and time t of a uniforr electric transmission line with resistance R inductance L, capacity C and leakag conductance G per unit length. Find th equations for I and V in the following cases :

(i) Lossless transmission line ($R = G = 0$),

(ii) Ideal submarine cable ($L = G = 0$),

(iii) Heaviside's distortion less line $\frac{R}{L} = \frac{G}{C}$ Constant $= k$.

Or

(b) Classify the equations and reduce int canonical form.

(i) $yu_{xx} - xu_{yy} = 0$, $x > 0$, $y > 0$;

(ii) $u_{xx} - (\sec^4 x)u_{yy} = 0$;

(iii) $u_{xx} + 6u_{xy} + 9u_{yy} + 3yu_y = 0$.

22. (a) Solve $xu_{xx} - x^3u_{yy} - u_x = 0$, $x \neq 0$, $u(x, y) = f(y)$

on $y - \frac{x^2}{2} = 0$ for $0 \leq y \leq 2$,

$u(x, y) = g(y)$ on $y + \frac{x^2}{2} = 4$ for $2 \leq y \leq 4$.

Or

(b) Find the solution of the characteristics initial value problem.

$y^3u_{xx} - yu_{yy} + u_y = 0$,

$u(x, y) = f(x)$ on $x + \frac{y^2}{2} = 4$ for $2 \leq x \leq 4$,

$u(x, y) = g(x)$ on $x - \frac{y^2}{2} = 0$ for $0 \leq x \leq 2$, with

$f(2) = g(2)$.

23. (a) State and prove uniqueness theorem of the vibrating string problem.

Or

(b) Find the temperature distribution in a rod length l, the faces are insulated and the initial temperature distribution is given by $x(l - x)$.

24. (a) State and prove mean value theorem.

Or

(b) Solve the Dirichilet problem.

$$\nabla^2 u = -2y\,,\; 0 < x < 1,\; 0 < y < 1$$

$$u(0,\, y) = 0\,,\; u(1,\, y) = 0,\; 0 \le y \le 1$$

$$u(x,\, 0) = 0\,,\; u(x,\, 1) = 0\,,\; 0 \le x \le 1$$

25. (a) Solve the boundary value problem

$$\frac{1}{r}\frac{\partial G}{\partial r}\left(\frac{r\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0\,,\; r \ge 0\,,\; z > 0\,,$$

$$\frac{\partial u}{\partial z} = \begin{cases} 0, \; r > a,\; z = 0 \\ c,\; r < a,\; z = 0,\; c = \text{constant.} \end{cases}$$

Or

(b) Prove that Greens function is symmetric.